NAME ________________________ CLASS ____________ DATE ____________

Reteaching
9.4 Ellipses

◆ **Skill A** Writing an equation for a graphed ellipse

Recall Ellipse with a horizontal major axis: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

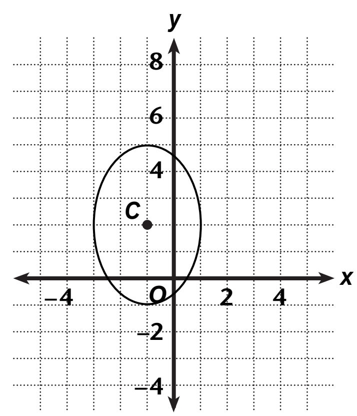
Ellipse with a vertical major axis: $\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$

where $a^2 > b^2$

In both cases, the center is at (h, k).

◆ **Example**

Write the standard equation for the ellipse shown at right.



◆ **Solution**

The center is at $(-1, 2)$, so $h = -1$ and $k = 2$.

The major axis is 6 units long, so $a = \frac{6}{2} = 3$.

The minor axis is 4 units long, so $b = \frac{4}{2} = 2$.

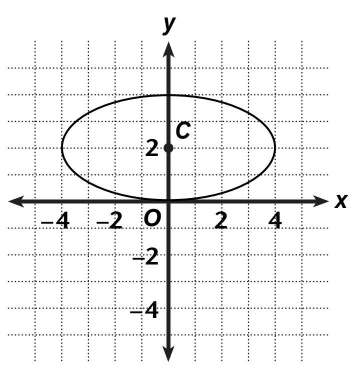
Since the major axis is vertical:

$$\frac{(x-(-1))^2}{2^2} + \frac{(y-2)^2}{3^2} = 1$$

The equation is $\frac{(x+1)^2}{4} + \frac{(y-2)^2}{9} = 1$.

Write the standard equation for each ellipse below.

1.



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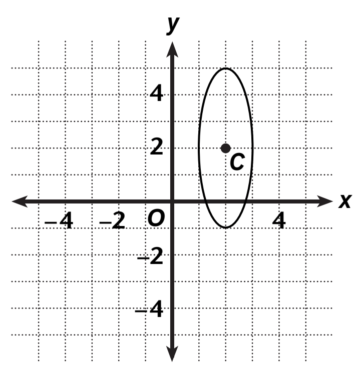
2.



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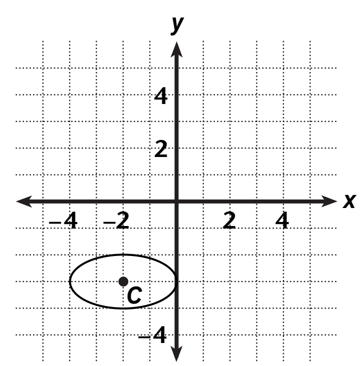
3.



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◆ **Skill B** Identifying the center, foci, vertices, and co-vertices of an ellipse

Recall The foci are on the major axis at a distance c from the center, where $c = \sqrt{a^2 - b^2}$.

◆ **Example**

Find the coordinates of the center, foci, vertices, and co-vertices for the ellipse $x^2 + 9y^2 - 4x + 18y + 4 = 0$.

◆ **Solution**

$x^2 - 4x + \_ + 9(y^2 + 2y + \_) = -4$

$x^2 - 4x + 4 + 9(y^2 + 2y + 1) = -4 + 4 + 9 \cdot 1$ *Complete the squares.*

$(x - 2)^2 + 9(y + 1)^2 = 9$

$\frac{(x-2)^2}{9} + \frac{(y+1)^2}{1} = 1$ *Standard Form*

$h = 2,\ k = -1,\ a = 3,\ b = 1,\ c = \sqrt{3^2 - 1^2} = \sqrt{8}$ *(Recall that $a^2 > b^2$.)*

Therefore, the center is at (2, −1).
Since a^2 is in the x-term, the major axis is **horizontal.**
The foci are at a **horizontal** distance c from the center:

$$(2 + \sqrt{8}, -1) \approx (4.8, -1) \text{ and } (2 - \sqrt{8}, -1) \approx (-0.8, -1).$$

The vertices are at a **horizontal** distance a from the center: (−1, −1) and (5, −1).
The co-vertices are at a **vertical** distance b from the center: (2, 0) and (2, −2).

Find the coordinates of the center, foci, vertices, and co-vertices of each ellipse.

4. $\frac{x^2}{16} + \frac{(y-2)^2}{36} = 1$

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5. $9x^2 + 25y^2 = 225$

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◆ **Skill C** Graphing an ellipse from its equation

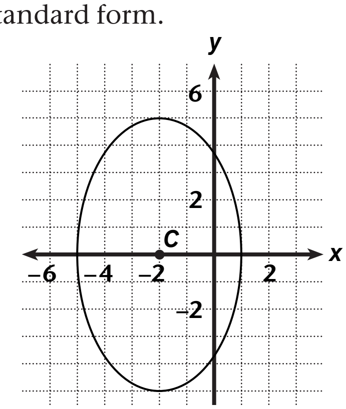
Recall To sketch a graph of any conic, write its equation in standard form.

◆ **Example**

Sketch a graph of $\frac{(x+2)^2}{9} + \frac{y^2}{25} = 1$.

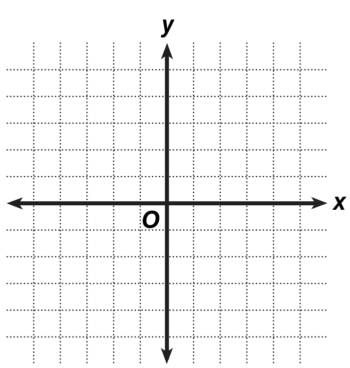
◆ **Solution**

The center is at (−2, 0). Since 9 is in the x-term, mark points at a distance $\sqrt{9} = 3$ **horizontally** from the center; (−5, 0) and (1, 0) are the vertices.
Since 25 is in the y-term, mark points at a distance $\sqrt{25} = 5$ **vertically** from the center; (−2, 5) and (−2, −5) are the co-vertices.



Sketch the graph of each ellipse.

6. $\frac{x^2}{25} + \frac{y^2}{9} = 1$



7. $25x^2 + (y + 1)^2 = 25$